The nonlocal abstract Schrödinger equations and applications

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This talk, that I would like to state today, devoted to the existence, uniqueness and L^p-regularity properties of Cauchy problem for nonlocal Schrodinger equations. The existence and regularity properties of solutions of Cauchy problem for Schrödinger equations (SE) studied e.g in $[1-4]$ and the references therein. The construction of general solutions of nonlocal SE were studied e.g. in [5-7]. Also, the existence and uniqueness of solutions of Cauchy problem for abstract SE were investigated in In [8, 9].

Here, the Cauchy problem for linear and nonlinear nonlocal Schrödinger equations are studied. The equation involves a convolution integral operators with a general kernel operator functions whose Fourier transform are operator functions defined in a Banach space E together with some growth conditions. By assuming enough smoothness on the initial data and the operator functions, the local and global existence and uniqueness of solutions are established. We can obtain a different classes of nonlocal Schrödinger equations by choosing the space E and linear operators, which occur in a wide variety of physical systems

The aim here, is to study the existence, uniqueness and regularity properties of solution of the initial value problem (IVP) for nonlocal nonlinear Schrödinger equatıon (NSE),

$$i\partial_t u + a\Delta u + A * u = B * f(u), \; t \in (0, T), \; x \in \mathbb{R}^n, \tag{1.1}$$

$$u(x, 0) = \varphi(x) \text{ for a.e. } x \in \mathbb{R}^n, \tag{1.2}$$

where $A = A(x)$, $B = B(x)$ are linear and nonlinear operator functions in a Hilbert space H, respectively, a is a complex number, $T \in (0, \infty]$, $f(u)$ is a given nonlinear function and $\varphi(x)$ is a given $E-$valued functions.

The method of proofs here, naturally differs from those used in previous works. Indeed, since the problem includes an abstract operator in the leading part and the problem is considered in E-valued L^p-spaces, we need some extra mathematics tools for deriving considered conclusions. For this reason, in the proof we use modern analysis tools like the following: (1) operator-valued Fourier multiplier theorems in abstract L^p spaces; (2) Embedding and trace theorems in Banach space valued Sobolev-Lions and Besov-Lions spaces; (3) Theory of semigroups of linear operators in Banach spaces; (4) Perturbation theory of operators;(5) Interpolation of Banach Spaces, and etc.